# Nature has no faithful mathematical representation

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June 4, 2012

*Abstract:* Much of modern theoretical physics assumes that the true nature of reality is mathematics. This is a great mistake. The assumption underlies most of the paradoxes of quantum mechanics, and has no empirical justification. Accepting that the assumption is wrong will allow physics and mathematics to progress as distinct disciplines.

Submitted to the FQXi.org 2012 Essay Contest answering: “Questioning the Foundations: Which of Our Basic Physical assumptions are Wrong?”

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Mathematics dominates theoretical physics, and underlies the deepest realities of nature. It is “unreasonably effective” in the words of E. Wigner. It is widely believed that the most fundamental objects of physics will be perfectly describable by mathematical structures. The structures might be variants of quantum field theory, or string theory, or supergravity, or some other unified field theory, but they will be given precisely by mathematical constants, formulas, equations, and other structures. I believe that this is a profound mistake.

Concepts of physics like mass, electricity, gravity, and electrons can be represented by mathematical structures. That is what the formulas in physics books are all about. A representation is *faithful* if it perfectly characterizes the physics. In particular, it must allow calculations that predict physical outcomes to as many decimal places as desired.

A faithful representation of the elementary particles (quarks, leptons, and bosons) is the holy grail of theoretical physics. I believe that there is no such thing, and that it is foolish to look for one.

## The mathematical reality assumption

Theoretical physicists commonly assume a faithful mathematical representation of nature, even if they do not admit it. They use the term “reality” to mean not just a physical reality, but also a linked mathematical reality that perfectly matches the physics.

The 2011 FQXi essay contest asked, “Is Reality Digital or Analog?” The answers accepted the premise that reality had to be one or the other, and no one admitted the possibility that it might be neither because both are mathematical.

A review of *The Grand Design*, a 2010 book by S. Hawking and L. Mlodinow, says, “He spends a lot of pages reviewing current physical theories but never mentions the one glaring feature they all share: Every modern physical theory, taken literally, predicts that our universe is a mathematical object. ... According to modern physics, everything is made of math.”

A review of *Dreams of a Final Theory,* a 1992 book by S. Weinberg, says that he "writes with great hope and clarity about the possibility that science can find a universal theory uniting the laws of nature into a single statement that is mathematically, philosophically, and aesthetically complete."

Rarely is the assumption explicit. MIT cosmologist M. Tegmark assumes it in his mathematical universe hypothesis. He says that our external physical reality is a mathematical structure. He goes further and says that all structures that exist mathematically also exist physically. His hypothesis is controversial, but it is taken for granted that our universe is an example of a mathematical structure.

Arguments for the many-worlds interpretation of quantum mechanics are almost entirely based on beliefs similar to Tegmark's. That is, some people believe that the mathematical possibilities must be the same as the physical possibilities, even if those possibilities cannot be observed.

All discussions of the alleged incompatibility between general relativity and quantum mechanics assume a mathematical reality. There is no foreseeable experiment that is capable of showing any such inconsistency. The only reason anyone says that these theories are incompatible is that it is hard to construct a faithful mathematical representation of both.

Wigner subscribed to an interpretation of quantum mechanics requiring a conscious observer to collapse the wave function. For those who say that this is too mystical, they nearly always require some sort of objective reality defined by mathematical structures.

Faith in the power and reality of mathematics has a long and colorful history. The Pythagoreans in 500 BC believed that numbers constituted the true nature of things. Galileo wrote in 1623 that the physics of the universe was "written in the language of mathematics". The idea is so entrenched today that hardly anyone distinguishes between scientific realism and some sort of mathematical idealization of the world.

S. Laplace proposed in 1814 that if every atom in the universe could be precisely mathematized, then the future could be predicted with deterministic certainty like a giant clockwork. Others have since argued that the universe is nondeterministic, based on formulas for quantum mechanics. Either way, any opinion on determinism is based on a mathematical reality assumption, and even the slightest imperfection invalidates the conclusion.

There are many proposals for a theory of everything, from string theory to various other unified field theories, and they nearly always start with the implicit premise that mathematics is the ultimate reality. These theories are not testable against any physical observations, so they are judged solely by how well they match our prejudices about what that ultimate mathematical reality ought to be. But if you drop the mathematical premise, there is nothing scientific about these theories.

## Complexity of mathematics

Mathematical models of physics can be extraordinarily complex, but even simple models are complex if you break them down into first principles. Even the simplest electron in isolation is too complicated to be fully described by mathematical structures. If it were a classical (point) particle, then it could be faithfully represented by its position, velocity, mass, electric charge, and maybe its spin (angular momentum). This is impossible for a quantum particle, because of the uncertainty principle.

A quantum electron has a certain mass, charge, and spin. Add a wave function for the position and momentum probability distribution. You also need quantum field theory with infinite mass and charge being renormalized to observable finite values, and it is still inadequate. There is no known mathematical model that fully describes an electron.

Even if an electron could be reduced to a few differential equations, or even a few real numbers, such a description is not so simple if you consider first principles. In mathematics, those principles are symbolic logic and axiomatic set theory (ZFC).

You might think of a real number as the simple outcome of a measurement, such as measuring a length with a meter stick. But that is just a crude approximation. If you want to take seriously the idea of a true underlying reality, then you need a true real number. Mathematically, real numbers can be rational or irrational. It is impossible to say whether a given measurement is rational or irrational. Mathematically, real numbers have infinite precision, and physically they are just approximations to measurements.

The rational numbers are good enough for measurements, but surely the irrationals are also needed for a physical reality. Some irrational numbers are so complicated that no computer can generate their decimal expansions. Some are not even definable in ZFC. Are these going to be allowed as part of the strict mathematical definition of an electron? No one can answer that.

The point here is that physics has intuitions about numbers that are adequate for measurements and approximations, but not for faithful mathematical representations of an electron or any other physical object. It seems unlikely that mathematical structures would be suitable for a true physical reality.

## Failure of hidden variables

Quantum mechanics has a long history of attempts to find faithful mathematical models of reality. All such attempts have failed decisively, both theoretically and experimentally.

The fundamental premise of quantum mechanics is: “The present paper seeks to establish a basis for theoretical quantum mechanics founded exclusively upon relationships between quantities which in principle are observable.” That is how W. Heisenberg began his famous 1925 paper.

But the quantum skeptics never accepted this premise. To them, quantum mechanics was riddled with uncertainties, and maybe those uncertainties were caused by an overly-restricted view of the physical variables.

By 1926, it was clear that Heisenberg uncertainties were essential to quantum mechanics. His approach was vigorously defended in Copenhagen by N. Bohr who said, “There is no quantum world. There is only an abstract physical description. It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature…”. J. von Neumann's 1932 textbook considered the possibility that the uncertainties could be eliminated by a hidden variable theory, and argued that no such theory was possible. Hidden variables are physical characteristics, like position, momentum, or spin, which determine the nature of something, but which are not directly observable. A. Einstein coauthored a 1935 paper arguing for such a theory anyway.

You can think of an electron wave function as telling you the probability that the electron will be found in a particular region. But if you think of the electron as a point particle, and the wave function as just a probability distribution for the position of that point particle, then you will run into trouble because Heisenberg uncertainty says that the electron cannot be a point particle.

D. Bohm proposed a hidden variable theory in 1952, but it violates locality and causality, and only works in trivial examples. J.S. Bell showed in 1964 that certain hidden variable theories make predictions that are contrary to quantum mechanics. While a lot of work has gone into this issue, most physicists consider the Bell test experiments to be the definitive proof that the hidden variable theories are impossible.

And yet interest in hidden variables continues unabated. A recent paper on the subject was submitted by M.F. Pusey, J. Barrett, and T. Rudolph under the title “The quantum state cannot be interpreted statistically”, and published in 2012 under the title, “On the reality of the quantum state”. Following von Neumann, Bell, and others, it shows again that certain hidden variable theories are incompatible with quantum mechanics. The paper generated a lot of excitement even though it merely affirmed the generally accepted view of 80 years ago.

It is rare in science for an 80-year-old theory to be so relentlessly challenged by theorists, and yet be so accurately confirmed by experiment. Does quantum mechanics have some flaw, or do the challengers have some conceptual misunderstanding? Why are physicists so fond of quoting R.P. Feynman and saying that no one understands quantum mechanics?

Even those who have given up on hidden variables have not given up on the search for a mathematical true reality for quantum mechanics. They say that the wave function is all there is to reality. Since the symbol $\psi$ (psi) is used for the wave function, and ontology is the metaphysical study of what really exists, they say physics is *psi-ontic*. The trouble with this view is that the wave function is not directly observable, and it becomes very difficult to understand what is meant by the measurement of an entangled particle.

Quantum mechanics treats interacting particles as an irreducible ensemble, and they can continue to be entangled even after they are separated. If two particles are entangled, and you measure one, then you immediately get information about the other. If quantum mechanics is psi-ontic, then measuring a particle causes a mysterious action-at-a-distance that is hard to understand because it violates our intuitions about causality. The philosopher K. Popper pointed this out as early as 1934, and proposed an experiment to test the concept.

The traditional Copenhagen interpretation of quantum mechanics is that the wave function is *psi-epistemic*, meaning that it represents our knowledge about the physical quantum state. There is no problem with that view, as long as you do not try to identify that state with hidden variables. In other words, it is the faithful mathematical representation assumption that leads to all sorts of contradictions and paradoxes.

There can be no hidden variables for quantum mechanics for the following reason. Quantum observables do not commute. That is the algebraic way to say that measuring two variables depends on which is measured first, and why observables like position and momentum cannot be measured at the same time. You can measure position and then momentum, or you can measure momentum and then position. The Heisenberg uncertainty principle says that any such attempts result in a discrepancy on the order of *h-bar*, Planck's constant. The constant is small, but quite easily measurable in many experiments.

It is also a mistake to say that the wave function represents probabilities. Probability is a mathematical concept and is not directly observed. One can use the theory to make statistical predictions, but the quantum state cannot be a literal probability because the wave function has negative and complex values.

Quantum mechanics does have statistical interpretations. All scientific theories have statistical interpretations. If a theory makes numerical predictions, and is tested with numerical measurements, then it is validated with statistical inferences. In this respect, quantum mechanics is no more or less probabilistic or statistical than classical mechanics or any other scientific theory.

The appeal of the hidden variables lies in the fact that a core principle of quantum mechanics is the idea that observations require projections to a lower dimension. That makes it tempting to believe that the full story lies in some higher dimensional space, and that we can unlock all the mysteries once we mathematize that higher dimensional space. Maybe the observables could somehow be replaced with other variables that commute and make all of the Heisenberg uncertainties disappear. But all such theories are destroyed by the fact that the non-commutation is experimentally confirmed.

The idea was described by the ancient Greek philosopher Socrates in about 400 BC, and written by Plato as the *Allegory of the Cave*. People in the cave see shadows, and do not appreciate the 3-D nature of the objects causing the shadows. They are seeing a 2-D projection of 3-D objects.

A photograph is also a 2-D projection of a 3-D scene. A measurement with a meter stick is a 1-D projection. Other observations can also be viewed as projections of some more complex reality.

Quantum mechanics is the first theory to truly take the cave allegory seriously. It has a theory for how observations correspond to projections, without ever trying to explain what is outside the cave. The theory concocts representations of reality, but is never sure about what the reality is.

Plato's student Aristotle is considered the father of science because he was concerned with what he could observe, as opposed to Plato's ideals. Platonism is a great way to do mathematics, but a lousy way to do physics.

## Quantum paradoxes

The most puzzling quantum experiments are the double-slit experiment and the spin measurement of entangled particles. Quantum mechanics predicts these outcomes without difficulty, but these experiments have been described as impossible to understand or as proof that there is no reality.

The double-slit experiment has a light beam or other beam going through two slits, and forming an interference pattern on the other side. If light is a wave, then there nothing surprising about the experiment. The puzzle occurs if you assume that the light consists of photons, that the photons must go through one slit or the other, and that a photon going through one slit has nothing to do with a photon going through the other. But all of those assumptions are like hidden variables in that they attribute unobserved mathematical properties to the light.

Some of the Bell test experiments are based on the spin measurement of entangled particles. It is possible to prepare two electrons to be in equal and opposite states, so that a measurement of one says a lot about the other. If you think of the electrons as physical objects, then there is no problem. The puzzle occurs when you attribute to the electron some idealized mathematical model where hidden variables predict the measurement.

In each case, the root of the paradox is not quantum mechanics or objectivity or reality or consciousness, but the unnecessary assumption of an underlying strictly mathematical reality.

Instead, textbooks and articles commonly draw faulty conclusions from these crucial experiments. Some say that there is instantaneous or nonlocal action-at-a-distance. Such fanciful claims should be regarded like claims of astrology or psychic phenomena. They are contrary to everything we know about the world, as well as to all of our scientific methodologies. If we had some convincing evidence of nonlocality, we would have to accept it, but we do not. We only have evidence that various mathematical hidden variable models would have to be nonlocal, and those models do not work very well anyway.

Another faulty conclusion is that the world is probabilistic or nondeterministic. The double-slit experiment can be slowed down to where it appears that an individual photon is randomly choosing a slit and then randomly choosing how to interfere with itself. If you measure the spin of one entangled electron and then the other, the outcome seems to follow probability formulas that depend on the orientation of the equipment and that resist simplification. But the experiments only show that the outcomes are not determined by simple hidden variable models, and say nothing about determinism from non-mathematical causes.

Most bizarre of all, it is often claimed that these experiments disprove local realism, or that there is no reality in nature. There is no evidence against physical reality; there is only evidence against physical and mathematical realities being identical.

The experiments are confusing if you think about an electron or photon as a point particle that is spinning and localized in space and time. One of the primary lessons of quantum mechanics is that there is no such thing as such a particle. The theory uses the *quantum*, which has both wave and particle properties.

When a photon passes through a double-slit, it is really a quantum of light that goes through both slits, not a probabilistic particle that is two places at once. The trouble only arises when you assume that the quantum has a hidden location variable that puts the quantum in one slit or the other. There is no such mathematical variable. The quantum is a physical unit of light that cannot be so neatly divided mathematically.

Likewise, an entangled electron pair is a mathematically indivisible quantum. The electrons can be physically separated, and then separately measured. Quantum mechanics says a lot about those measurements, but it does not provide a separated mathematical representation of the electrons while they are still entangled. All attempts at such a representation have failed.

Arguments about determinism depend on being able to perfectly replicate experimental conditions, including any hidden variables. If all of those conditions can be defined with numbers, then we have a notion of precisely redoing an experiment to see whether we get the same result. Otherwise, we do not.

The quantum is real, physical, and causal. It just isn't reducible to purely mathematical components, and we have no way of knowing whether the question of determinism even makes any sense.

## Lessons from relativity

The discovery of quantum mechanics followed on the heels of the spectacular success of relativity, and some lessons were learned from that history. The crucial experiment was by Michelson-Morley in 1887, showing that the speed of light was the same in different frames of reference. G. FitzGerald was the first to make the logical deduction from the apparent contradiction, in 1889, by saying, "I would suggest that almost the only hypothesis that can reconcile this opposition is that the length of material bodies changes …". H. Lorentz made a similar deduction and showed how the changes in space and time could be reconciled with the equations for electromagnetism.

The lesson here is that logic and experiment can drive you to give up a cherished concept. Likewise, logic and experiment have shown the impossibility of hidden variable and psi-ontic theories. It is time to accept the non-mathematical nature of reality, just as it was time to accept the non-Euclidean geometry of spacetime when H. Poincare proposed it in 1905.

Another driving force behind relativity was Poincare's belief in local causality. He explained in his 1902 book, *Science and Hypothesis*, that causality is what allows mechanics to be described by differential equations, and that the notion was essential for science. He lectured in 1904 about how matter and communications signals would have to be limited by the speed of light. And he invented four-dimensional spacetime in 1905 in order to explain how gravity could propagate as waves at the speed of light, instead of by Newtonian action-at-a-distance.

The causality lesson is that locality of cause and effect is an integral part of modern physics, and there is no going back to action-at-a-distance. Interpretations of quantum mechanics that violate local causality have been dead-ends with no theoretical or experimental support.

Another lesson from relativity was that the notion of reality was a distraction. If you asked whether or not the moving measuring rods were really contracted, you could not get a good answer. Lorentz did show that the contraction could be accounted for by distortions in the electromagnetic fields that hold together the molecules of a rigid body, and that analysis is valid. But the preferred view is to ignore the reality of the rigid body, and treat relativity as a theory about "something which would be due to our methods of measurement." Likewise in quantum mechanics, it is better to ignore the reality of assigning numerical values to objects, and instead focus on the measurements.

The history of relativity is that for decades there were many skeptics who refused to accept the more startling conclusions, and who pointed to various paradoxes and alleged inconsistencies, in order to reject the theory. The situation is much worse in quantum mechanics, as even today prominent physicists give popular accounts of the theory that are contrary to established understandings.

## Folly of quantum computing

The most conspicuous consequence of a faithful mathematical representation assumption is quantum computing. A quantum computer, if one exists, could outperform a Turing machine on certain types of problems. While many experiments of many kinds have confirmed many aspects of quantum mechanics, none have demonstrated computation superior to a Turing machine.

An ordinary (Turing machine) computer typically has a 64-bit register that can hold a 64-bit integer. A bit is a 0 or 1. A quantum computer might have 64 *qubits*, where each qubit can be a 0 or a 1 or any probability in between. Assuming that the qubits are exactly linked to some underlying mathematical reality, the quantum computer can do a probabilistic calculation that somehow represents all 64-bit integers at once.

The ordinary computer can do a long complex calculation, and it will use electronic components designed with quantum mechanics, but it does not assume any mathematical reality. Every bit operation is rounded to a 0 or 1, and the arithmetic is layered on top of that with logic gates. But quantum computers do the calculation within a hypothesized mathematical structure in which qubits are identified with probabilities that could be irrational, negative, or even imaginary numbers, and that are linked mathematically to other qubits.

The quantum computer scientists are always saying that their theory is a consequence of what we know about quantum mechanics. It seems to me that it is the opposite. The lesson of $20^{th}$ century quantum mechanics is that we do not have faithful representations of multi-qubit states or any other complex state.

The theory behind quantum computation is a gigantic extrapolation from quantum mechanics. It assumes that quantum states are perfect mathematical representations of reality, and that they can be added, transformed, and projected repeatedly with great mathematical precision. If the quantum states are really mathematical structures and if those mathematical operations can be made physically, then quantum computing seems possible.

The whole spirit of quantum computing assumes a hidden mathematical reality that is far beyond Heisenberg's vision of sticking to observables.

Quantum mechanics is wildly successful, and if you take the mathematics literally then some interpretations say that on the smallest scales the math predicts that virtual particles are created, that energy is not conserved, that particles go backwards in time, that particles can be in two places at once, that there is nonlocal action-at-a-distance, that there are entangled qubits, and that computation can exceed the Turing limits. And yet none of this is ever observed on a macroscopic level.

Quantum computing is sometimes considered the natural progression in research on entanglement, quantum cryptography, teleportation, and many-worlds interpretation. But all of those areas are only made mysterious by dubious claims about an underlying mathematical reality. None of the experiments show anything startling, unless you are somehow persuaded of those dubious claims, and there is no proof that anything useful will come out of this work.

## Positivism

A positivist would not accept that physics has any faithful mathematical representations without compelling evidence. There is none. Quantum mechanics is a positivist theory. Positivism has fallen out of favor in part because some of its early adherents refused to accept the reality of atoms. Atoms certainly exist in the sense that a carbon atom can be distinguished from an oxygen atom, but our intuition about the reality of atoms is crude and our mathematical models are imprecise.

I don’t expect others to adopt my positivist philosophy. But when they theorize with an assumption that is contrary to much of $20^{th}$ century physics, they should at least make that assumption explicit. The faithful mathematical representation hypothesis is such an assumption, and it might be completely wrong.

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