Positivist perspective on predictability

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Mar. 18, 2020

Abstract: Mathematicians rely on rigorous proofs to know what is true. By analogy, there are positivist scientists who similarly confine their work to what can be empirically established. Under such views, physicists need not hope to predict everything, just as mathematicians do not hope to decide everything.

Submitted to the FQXi.org 2020 Essay Contest on: "Undecidability, Uncomputability, and Unpredictability."

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Introduction

The late Freeman Dyson believed that Physics was open-ended, and would never be completely mastered. His view was somewhat contrary to that of the clockwork universe, where the world is like a giant machine that is understandable and predictable, if all the parts are understood. Those with the clockwork view acknowledge that it might take a deity or a demon, such as Laplace's demon, to master all the predictions, but they insist that it is possible in principle.

Dyson believed that principles like Goedel's incompleteness theorem might make such mastery impossible. I take a somewhat more limited view of what math and science can do, and argue that the whole question is meaningless.

Positivism

Logical positivism was a popular philosophy of the first half of the XX century, but is considered dead by today's philosophers. It is historically important as it helped shape the thinking behind quantum mechanics and many other advances. It was so important that it is not clear that most scientists would accept its death today.

I believe that it should not have died, and that it is superior to the philosophies that replaced it. However, my purpose here is not to convince you of that. It is only to explain how an extreme positivist views certain scientific issues, so that you can appreciate another perspective.

The nature of positivism is to emphasize what can be known for sure, and to avoid speculation about other matters. This means having little to say about metaphysical discussions about Laplace's demon.

The positivist might say, "I cannot prove or disprove Laplace's demon. If you want to believe in it without proof, that is like having a religous belief or superstition, but I have nothing to say because it is outside the realm of knowledge that I recognize."

Critics of positivism get frustrated with this argument, as it seems to reject many issues without substantively engaging with them. They mostly argue that positivism is some sort of cop-out.

The mysterious real numbers

The ordinary real numbers are naturally intuitive, as they are the result of every measurement. You could say that all of science collects real numbers about nature, and emits other real numbers that say something about the world.

Even Amazon tribes with limited counting practices seem to have a primitive analog number sense. Science is all about real numbers.

Likewise with Mathematics. The birth of mathematical analysis is intertwined with coming to a rigorous understanding of the reals. Real numbers are typically defined as limits of sequences of rational numbers, and the concept of a limit is a subtle one that took centuries to perfect.

Once real numbers became rigorously defined, it was soon obvious that they can never been fully understood. There are uncountably many of them. Measure theory was invented to measure them, but the theory has to carefully avoid oddball paradoxes about non-measurable sets. The continuum hypothesis could not be resolved.

Furthermore, even individual real numbers can be very strange. Only countably many can be generated by Turing machines, so most of them are non-computable.

One real number could encode all the world's knowledge. Or all possible knowledge. Even more strangely, one real number could be an oracle that could used to generate non-computable numbers, and solving computational problems that are otherwise impossible.

If these numbers could be found in nature, along with methods for extracting arbitrarily many digits, then we would have to revise what we know about the feasibility of computation.

Finitary methods

The surest form of knowledge is the mathematical proof. It was invented by the ancient Greeks, and not by anyone else. The method was demonstrated in Euclid's Elements, and is still taught in high schools today. Other geometries have since been discovered, but the truths of Euclidean geometry are immortal.

Mathematical proofs are *finitary*. The theorems are expressed with a finite set of symbols. The number of steps is finite. Finitely many axioms are used.

Consider the theorem that any infinite set of real numbers between 0 and 1 contains a convergent sequence. This is a statement about infinite sets. To understand it, you need to know that there are infinitely many real numbers, and what it means to take the limit of an infinite sequence. In spite of these infinities, the proofs are entirely finitary.

Finitary proofs from a given axiom system cannot resolve all questions about arithmetic. Goedel proved that. Apparently this discovery was a disappointment to many mathematicians at the time. It means that we cannot just write a computer program to settle all mathematical questions.

However, it takes a lot of arrogance to think that one might find an algorithm to answer all mathematical questions anyway. We simply have to recognize that our knowledge is always incomplete, and we cannot expect to answer all questions.

Continuum hypothesis not meaningful

The continuum hypothesis says that there are no cardinal numbers between the integers and the real numbers. That is, every subset of the reals is either countable, or can be put in a one-to-one correspondence with the reals.

A common view is that the continuum hypothesis is either true or false, but we don't know which because our axioms are not powerful enough.

The more positivist view is that it is meaningless to discuss the truth of the continuum hypothesis. The infinities do not even exist, except in a carefully qualified sense. We can prove and disprove statements about infinities, but those proofs are really finitary arguments that have an interpretation involving infinities.

If a theorem specifies a set, then that set is defined by a finite set of symbols. That is how mathematics works. We can talk about the existence of some exotic set, but such existence is always a shorthand for a finitary argument.

The continuum hypothesis was such an important problem that the mathematician David Hilbert listed it first on his famous 1900 list of open problems. But he also said, "The infinite is nowhere to be found in reality, no matter what experiences, observations, and knowledge are appealed to."

Not all mathematicians even accept the reality of infinities in mathematics. C.F. Gauss said that "infinity is merely a way of speaking". Mathematicians use infinities all the time, but the usage is really a way of speaking about finitary arguments.

Positivist quantum mechanics

The theory of quantum mechanics raises a lot of questions that have no easy answers. Is an electron a particle or a wave? What can we say about its precise position and momentum? How can we describe the state of an electron without referencing some distant entangled electron?

The theory was developed with a positivist philosophy of focusing on predicting observations. Other questions are not necessarily meaningful, and can be ignored.

Positivists do not believe in many-worlds theory, as there cannot be any proof of the existence of a parallel world that is not observable.

The many-worlds theory is one of several that try to deny Copenhagen collapses of the wave function. They say that the Schroedinger equation should allow predicting future wave functions from present wave functions. One trouble with that view is that the wave function is not directly an observable.

Sometimes you can observe an electron in an energy eigenstate, and essentially know what its wave function must be. But for more complex systems, the wave function is not really knowable. So even if you could predict future wave functions from present wavefunctions, it still would not solve the problem of getting future observables from present observables.

Positivist skepticism

It is widely accepted that the fine structure constant is a real number approximately equal to 1/137. Experiments have determined it to about ten decimal places. Is it possible that the true value is one of those exotic real numbers that encodes infinite amounts of knowledge?

It would be impractical to extract that knowledge, but that would not stop physicists from speculating about it. It is impossible for an outsider to observe anything inside the event horizon of a black hole, but there are a great many papers on the subject anyway.

R.P. Feynman wrote about the fine structure constant, “We know what kind of a dance to do experimentally to measure this number very accurately, but we don’t know what kind of dance to do on the computer to make this number come out, without putting it in secretly!” We don’t even know whether it is a number that a Turing machine can generate.

The fine structure constant is not really a real number. Saying that it is a real number is just a shorthand for saying that a certain range of numbers is consistent with experiments.

The extreme positivist has skepticism about attributing reality to any sort of infinity, whether it be the infinite extent of space or the infinite decimal expansion of some physical constant. Saying that space has infinite extent might be a useful shorthand for saying that there are no known barriers to traveling great distances, but that’s all.

Brian Greene likes to talk about various cosmological infinities, such as infinitely many universes, infinitely many doppelgangers of himself, parallel worlds, etc. To the positivist, these are all fairy tales. They don’t even really exist mathematically.

Over a century ago, positivists were skeptical about atoms and electrons. It seemed impossible for humans to directly observe atoms or electrons, so maybe they were just theoretical constructs.

In fact, thinking of electrons as particles leads to erroneous conclusions. Electrons have wave-like and particle-like properties.

You might think that electrons are observed as point particles, and therefore a positivist should regard them as point particles. But if an electron were a point particle, then it would have infinite mass density and infinite charge density. Quantum electrodynamics and renormalization theory have a way of dealing with such infinities, but we don’t know that the infinities are really there. No experiment has ever been able to probe the electron closely to see evidence of very large densities, and such an observation may be impossible.

The renormalization theory is really just a clever shorthand for dealing with very high concentrations of mass and charge.

Some physicists have an optimism that we will have a theory of everything, and that it will be able to make all future predictions, given accurate initial data. An extreme form of such thinking is the hope for a superdeterministic theory, where all possible physical variables are determined from the first instant of the big bang, and where all choices and randomness are mere illusions.

The positivist regards such theorizing as outside the realm of science. There does not appear to be any way to confirm any such theory. It is not even a goal that is worth working toward. Science is about gaining definite knowledge, and not inventing imaginary universes.

Not negativism

The XX century can be seen as an era when limits to human knowledge were discovered. Goedel showed that not all truths could be proved. Quantum mechanics showed that position and momentum could not be simultaneously measured. Turing found limits to machine computation. The discovery of chaos limited long-term weather predictions.

The optimist regards the glass of water as half-full, not half-empty.

Did previous scientists really believe that someday a computer could be programmed to determine all mathematical truths and predict all physical phenomena? I doubt it. That would require a belief in an extreme form of determinism, and a depressing view of humanity. We would all be pre-programmed robots. Some man's brilliant mathematical idea would be no better than memorized digits of pi. A computer could do it better.

My hunch is that those scientists believed that humans were better than just robots, and that there were limits to knowledge. Flavio Del Santo makes a convincing argument that some pre-quantum physicists did not regard classical mechanics as deterministic.

If I told them that in 2020 we would have useful 5-day weather forecasts, would they have argued it should be possible to forecast weather months or years in advance? I doubt it.

When quantum mechanics was discovered in the 1920s, it described physics on an atomic level, as previously not possible. Scientists learned that they could make amazingly precise predictions, and that there were fundamental uncertainties blocking other types of predictions. Which discovery was more surprising? My guess is that the ability to make precise atomic predictions was much more surprising.

Are there implications for our understanding of the relations between agency, intelligence, mind, and the physical world? I am not sure, but everything has its limits. The more we learn, the more we bump into limits of knowledge, and of what is possible. Trying to get past those limits can get us lost. Our greatest progress has been from sticking to what can be positively demonstrated, from either axioms or experiments.

Conclusion

Positivism is a legitimate philosophical view. Mathematics has a long tradition of sticking to what can be formally proved from axioms. Physics would be enriched by popularizing a similar view, so that we can more easily distinguish established knowledge from speculation.

I am not trying to persuade anyone to stop speculating about the interior of black holes, but to understand that positivists can reasonably argue that such theorizing has no known scientific value.

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